nature photonics

SUPPLEMENTARY INFORMATION

DOI: 10.1038/NPHOTON.2010.5

Qiang Lin, Jessie Rosenberg, Darrick Chang, Ryan Camacho,

Matt Eichenfield, Kerry J. Vahala and Oskar Painter*

Thomas J. Watson, Sr., Laboratory of Applied Physics,

California Institute of Technology, Pasadena, CA 91125

(Dated: December 16, 2009)

Abstract

We present in this supplement to our manuscript entitled, "Coherent mixing of mechanical excitations in nano-optomechanical structures," the detailed analysis related to mechanical mode mixing and renormalization.

PACS numbers:

*Electronic address: opainter@caltech.edu; URL: http://copilot.caltech.edu

DOI: 10.1038/NPHOTON.2010.5

I. INTRACAVITY FIELD IN THE PRESENCE OF OPTOMECHANICAL COUPLING

In the presence of optomechanical coupling, the optical field inside the cavity satisfies the following equation:

$$\frac{da}{dt} = (i\Delta_0 - \Gamma_t/2 - i g_{om} x_b) a + i\sqrt{\Gamma_e} A_{in}, \tag{S1}$$

where a is the optical field of the cavity mode, normalized such that $U = |a|^2$ represents the mode energy, and A_{in} is the input optical wave, normalized such that $P_{in} = |A_{in}|^2$ represents the input power. Γ_t is the photon decay rate for the loaded cavity and Γ_e is the photon escape rate associated with the external coupling. $\Delta_0 = \omega - \omega_0$ is the frequency detuning from the input wave to the cavity resonance. g_{om} is the optomechanical coupling coefficient associated with the optically bright mode, with a mechanical displacement given by x_b. In Eq. (S1), we have neglected the optomechanical coupling to the optically dark mode because of its negligible magnitude.

Well below the threshold of mechanical oscillation, the mechanical motion is generally small, and its impact on the intracavity optical field can be treated as a small perturbation. As a result, the intracavity field can be written as $a(t) \approx a_0(t) + \delta a(t)$, where a_0 is the cavity field in the absence of optomechanical coupling and δa is the perturbation induced by the mechanical motion. They satisfy the following two equations:

$$\frac{da_0}{dt} = (i\Delta_0 - \Gamma_t/2) a_0 + i\sqrt{\Gamma_e} A_{in}, \tag{S2}$$

$$\frac{d\delta a}{dt} = (i\Delta_0 - \Gamma_t/2)\delta a - i g_{om} x_b a_0. \tag{S3}$$

In the case of a continuous-wave input, Eq. (S2) leads to a steady state given by

$$a_0 = \frac{i\sqrt{\Gamma_e} A_{in}}{\Gamma_t/2 - i\Delta_0}, \tag{S4}$$

and Eq. (S3) provides a spectral response for the perturbed field amplitude of

$$\delta\tilde{a}(\Omega) = \frac{i g_{om} a_0 \tilde{x}_b(\Omega)}{i(\Delta_0 + \Omega) - \Gamma_t/2}, \tag{S5}$$

where $\delta\tilde{a}(\Omega)$ is the Fourier transform of $\delta a(t)$ defined as $\delta\tilde{a}(\Omega) = \int_{-\infty}^{+\infty} \delta a(t) e^{i\Omega t} dt$. Similarly, $\tilde{x}_b(\Omega)$ is the Fourier transform of $x_b(t)$.

II. THE POWER SPECTRAL DENSITY OF THE CAVITY TRANSMISSION

From the discussion in the previous section, the transmitted optical power from the cavity is given by

$$P_T = \left|A_{in} + i\sqrt{\Gamma_e} a\right|^2 \approx |A_0|^2 + i\sqrt{\Gamma_e}\,(A_0^* \delta a - A_0 \delta a^*)\,, \tag{S6}$$

where A_0 is the steady-state cavity transmission in the absence of optomechanical coupling. It is given by

$$A_0 = A_{in} \frac{(\Gamma_0 - \Gamma_e)/2 - i\Delta_0}{\Gamma_t/2 - i\Delta_0}, \tag{S7}$$

where Γ_0 is the photon decay rate of the intrinsic cavity. It is easy to show that the averaged cavity transmission is given by $\langle P_T \rangle = |A_0|^2$, as expected. By using Eqs. (S5), (S6), and (S7), we find the power fluctuations, $\delta P_T(t) \equiv P_T(t) - \langle P_T \rangle$, are given in the frequency domain by

$$\delta \widetilde{P}_T(\Omega) = \frac{i\Gamma_e P_{in} g_{om} \widetilde{x}_b(\Omega)}{(\Gamma_t/2)^2 + \Delta_0^2} \left[\frac{(\Gamma_0 - \Gamma_e)/2 + i\Delta_0}{\Gamma_t/2 - i(\Delta_0 + \Omega)} - \frac{(\Gamma_0 - \Gamma_e)/2 - i\Delta_0}{\Gamma_t/2 + i(\Delta_0 - \Omega)} \right], \tag{S8}$$

where $\delta \widetilde{P}_T(\Omega)$ is the Fourier transform of $\delta P_T(t)$. By using Eq. (S8), we obtain a power spectral density (PSD) for the cavity transmission of

$$S_P(\Omega) = g_{om}^2 P_{in}^2 S_{x_b}(\Omega) H(\Omega), \tag{S9}$$

where $S_{x_b}(\Omega)$ is the spectral intensity of the mechanical displacement for the optically bright mode which will be discussed in detail in the following sections. $H(\Omega)$ is the cavity transfer function defined as

$$H(\Omega) \equiv \frac{\Gamma_e^2}{\left[\Delta_0^2 + (\Gamma_t/2)^2\right]^2} \frac{4\Delta_0^2(\Gamma_0^2 + \Omega^2)}{\left[(\Delta_0 + \Omega)^2 + (\Gamma_t/2)^2\right]\left[(\Delta_0 - \Omega)^2 + (\Gamma_t/2)^2\right]}. \tag{S10}$$

In general, when compared with $S_{x_b}(\Omega)$, $H(\Omega)$ is a slowly varying function of Ω and can be well approximated by its value at the mechanical resonance: $H(\Omega) \approx H(\Omega'_{mb})$. Clearly then, the power spectral density of the cavity transmission is linearly proportional to the spectral intensity of the mechanical displacement of the optically bright mode.

III. THE MECHANICAL RESPONSE WITH MULTIPLE EXCITATION PATHWAYS

When the optically bright mode is coupled to an optically dark mode, the Hamiltonian for the coupled mechanical system is given by the general form:

$$\mathscr{H}_m = \frac{p_b^2}{2m_b} + \frac{1}{2}k_b x_b^2 + \frac{p_d^2}{2m_d} + \frac{1}{2}k_d x_d^2 + \kappa x_b x_d, \tag{S11}$$

 DOI: 10.1038/NPHOTON.2010.5

where x_j, p_j, k_j, and m_j ($j = b,d$) are the mechanical displacement, kinetic momentum, the spring constant, and the effective motional mass for the $j^{\rm th}$ mechanical mode, respectively, and κ represents the mechanical coupling between the bright and dark modes. The subscripts b and d denote the optically bright and optically dark modes, respectively. With this system Hamiltonian, including the optical gradient force on the optically bright mode and counting in the mechanical dissipation induced by the thermal mechanical reservoir, we obtain the equations of motion for the two mechanical modes:

$$\frac{d^2x_b}{dt^2} + \Gamma_{mb}\frac{dx_b}{dt} + \Omega_{mb}^2 x_b + \frac{\kappa}{m_b}x_d = \frac{F_b}{m_b} + \frac{F_o}{m_b}, \tag{S12}$$

$$\frac{d^2x_d}{dt^2} + \Gamma_{md}\frac{dx_d}{dt} + \Omega_{md}^2 x_d + \frac{\kappa}{m_d}x_b = \frac{F_d}{m_d}, \tag{S13}$$

where $\Omega_{mj}^2 \equiv \frac{k_j}{m_j}$ is the mechanical frequency for the $j^{\rm th}$ mode. F_j ($j = b,d$) represents the Langevin forces from the thermal reservoir actuating the Brownian motion, with the following statistical properties in the frequency domain:

$$\langle \widetilde{F}_i(\Omega_u)\widetilde{F}_j^*(\Omega_v)\rangle = 2m_i\Gamma_{mi}k_BT\delta_{ij}2\pi\delta(\Omega_u - \Omega_v), \tag{S14}$$

where $i,j = b,d$, T is the temperature and k_B is the Boltzmann constant. $\widetilde{F}_i(\Omega)$ is the Fourier transform of $F_i(t)$.

In Eq. (S12), $F_o = -\frac{g_{om}|a|^2}{\omega_0}$ represents the optical gradient force. From the previous section, we find that it is given by

$$F_o(t) = -\frac{g_{om}}{\omega_0}\left[|a_0|^2 + a_0^*\delta a(t) + a_0\delta a^*(t)\right]. \tag{S15}$$

The first term is a static term which only changes the equilibrium position of the mechanical motion. It can be removed simply by shifting the mechanical displacement to be centered at the new equilibrium position. Therefore, we neglect this term in the following discussion. The second and third terms provide the dynamic optomechanical coupling. From Eq. (S5), the gradient force is found to be given in the frequency domain by

$$\widetilde{F}_o(\Omega) \equiv f_o(\Omega)\widetilde{x}_b(\Omega) = -\frac{2g_{om}^2|a_0|^2\Delta_0\widetilde{x}_b(\Omega)}{\omega_0}\frac{\Delta_0^2 - \Omega^2 + (\Gamma_t/2)^2 + i\Gamma_t\Omega}{[(\Delta_0+\Omega)^2 + (\Gamma_t/2)^2][(\Delta_0-\Omega)^2 + (\Gamma_t/2)^2]}, \tag{S16}$$

which is linearly proportional to the mechanical displacement of the optically bright mode.

Equations (S12) and (S13) can be solved easily in the frequency domain, in which the two equations become

$$L_b(\Omega)\widetilde{x}_b + \frac{\kappa}{m_b}\widetilde{x}_d = \frac{\widetilde{F}_b}{m_b} + \frac{\widetilde{F}_o}{m_b}, \tag{S17}$$

$$L_d(\Omega)\widetilde{x}_d + \frac{\kappa}{m_d}\widetilde{x}_b = \frac{\widetilde{F}_d}{m_d}, \tag{S18}$$

where $L_j(\Omega) \equiv \Omega_{mj}^2 - \Omega^2 - i\Gamma_{mj}\Omega$ $(j = b, d)$. Substituting Eq. (S16) into Eq. (S17), we find that Eq. (S17) can be written in the simple form,

$$L_b(\Omega)\widetilde{x}_b + \frac{\kappa}{m_b}\widetilde{x}_d = \frac{\widetilde{F}_b}{m_b}, \tag{S19}$$

where $L_b(\Omega)$ is now defined with a new mechanical frequency Ω'_{mb} and energy decay rate Γ'_{mb} as

$$L_b(\Omega) = \Omega_{mb}^2 - \Omega^2 - i\Gamma_{mb}\Omega - \frac{f_o(\Omega)}{m_b} \equiv (\Omega'_{mb})^2 - \Omega^2 - i\Gamma'_{mb}\Omega, \tag{S20}$$

and the new Ω'_{mS} and Γ'_{mS} are given by

$$\begin{aligned} (\Omega'_{mb})^2 &\equiv \Omega_{mb}^2 + \frac{2g_{om}^2|a_0|^2\Delta_0}{m_b\omega_0}\frac{\Delta_0^2 - \Omega^2 + (\Gamma_t/2)^2}{[(\Delta_0+\Omega)^2 + (\Gamma_t/2)^2][(\Delta_0-\Omega)^2 + (\Gamma_t/2)^2]} \\ &\approx \Omega_{mb}^2 + \frac{2g_{om}^2|a_0|^2\Delta_0}{m_b\omega_0}\frac{\Delta_0^2 - \Omega_{mb}^2 + (\Gamma_t/2)^2}{[(\Delta_0+\Omega_{mb})^2 + (\Gamma_t/2)^2][(\Delta_0-\Omega_{mb})^2 + (\Gamma_t/2)^2]}, \end{aligned} \tag{S21}$$

$$\begin{aligned} \Gamma'_{mb} &\equiv \Gamma_{mb} - \frac{2g_{om}^2|a_0|^2\Gamma_t\Delta_0}{m_b\omega_0}\frac{1}{[(\Delta_0+\Omega)^2 + (\Gamma_t/2)^2][(\Delta_0-\Omega)^2 + (\Gamma_t/2)^2]} \\ &\approx \Gamma_{mb} - \frac{2g_{om}^2|a_0|^2\Gamma_t\Delta_0}{m_b\omega_0}\frac{1}{[(\Delta_0+\Omega_{mb})^2 + (\Gamma_t/2)^2][(\Delta_0-\Omega_{mb})^2 + (\Gamma_t/2)^2]}. \end{aligned} \tag{S22}$$

Clearly, the effect of the optical gradient force on the optically bright mode is primarily to change its mechanical frequency (the optical spring effect) and energy decay rate (mechanical amplification or damping).

Equations (S18) and (S19) can be solved easily to obtain the solution for the optically bright mode,

$$\widetilde{x}_b(\Omega) = \frac{\frac{\widetilde{F}_b(\Omega)}{m_b}L_d(\Omega) - \frac{\kappa}{m_b}\frac{\widetilde{F}_d(\Omega)}{m_d}}{L_b(\Omega)L_d(\Omega) - \eta^4}, \tag{S23}$$

where $\eta^4 \equiv \frac{\kappa^2}{m_b m_d}$ represents the mechanical coupling coefficient. By using Eq. (S14) and (S23), we obtain the spectral intensity of the mechanical displacement for the optically bright mode,

$$S_{x_b}(\Omega) = \frac{2k_BT}{m_b}\frac{\eta^4\Gamma_{md} + \Gamma_{mb}|L_d(\Omega)|^2}{|L_b(\Omega)L_d(\Omega) - \eta^4|^2}, \tag{S24}$$

where $L_b(\Omega)$ is given by Eq. (S20). The mechanical response given by Eq. (S24) is very similar to the atomic response in EIT.

DOI: 10.1038/NPHOTON.2010.5

As discussed in the previous section, the power spectral density (PSD) of the cavity transmission is linearly proportional to Eq. (S24). Equation (S24) together with (S9) is used to find the theoretical PSD shown in Fig. 3, by using an optomechanical coupling coefficient of $g_{om}/2\pi = 33$ GHz/nm and an effective mass of $m_b = 264$ pg for the flapping mode, both obtained from FEM simulations. The intrinsic and loaded optical quality factors of 1.07×10^6 and 0.7×10^6 are obtained from optical characterization of the cavity resonance, and are also given in the caption of Fig. 1 of the main text. The intrinsic mechanical frequencies and damping rates of the two modes (Ω_{mb}, Ω_{md}, Γ_{mb}, and Γ_{md}) are obtained from the experimentally recorded PSD of cavity transmission with a large laser-cavity detuning, as given in the main text. The mechanical coupling coefficient η is treated as a fitting parameter. Fitting of the PSDs results in $\eta/2\pi = 3.32$ MHz, indicating a strong internal coupling between the two mechanical modes. As shown clearly in Fig. 3d, f-h of the main text, our theory provides an excellent description of the observed phenomena.

IV. THE MECHANICAL RESPONSE WITH EXTERNAL OPTICAL EXCITATION

The previous section focuses on the case in which the mechanical excitations are primarily introduced by the thermal perturbations from the environmental reservoir. However, the mechanical motion can be excited more intensely through the optical force by modulating the incident optical wave. In this case, the input optical wave is composed of an intense CW beam together with a small modulation: $A_{in} = A_{in0} + \delta A(t)$. As a result, Eq. (S3) now becomes

$$\frac{d\delta a}{dt} = (i\Delta_0 - \Gamma_t/2)\delta a - i g_{om} x_b a_0 + i\sqrt{\Gamma_e}\delta A. \tag{S25}$$

This equation leads to the intracavity field modulation given in the frequency domain as:

$$\delta\widetilde{a}(\Omega) = \frac{i g_{om} a_0 \widetilde{x}_b(\Omega) - i\sqrt{\Gamma_e}\delta\widetilde{A}(\Omega)}{i(\Delta_0+\Omega) - \Gamma_t/2}, \tag{S26}$$

where $\delta\widetilde{A}(\Omega)$ is the Fourier transform of $\delta A(t)$. By use of this solution together with Eq. (S15), the gradient force now becomes

$$\widetilde{F}_o(\Omega) = f_o(\Omega)\widetilde{x}_b(\Omega) + \widetilde{F}_e(\Omega), \tag{S27}$$

where $f_o(\Omega)$ is given by Eq. (S16) and $\widetilde{F}_e(\Omega)$ represents the force component introduced by the input modulation. It is given by the following form:

$$\widetilde{F}_e(\Omega) = \frac{i\sqrt{\Gamma_e}g_{om}}{\omega_0}\left[\frac{a_0^*\delta\widetilde{A}(\Omega)}{i(\Delta+\Omega)-\Gamma_t/2} + \frac{a_0\delta\widetilde{A}^*(-\Omega)}{i(\Delta-\Omega)+\Gamma_t/2}\right]. \tag{S28}$$

In particular, in the sideband-unresolved regime, Eq. (S28) can be well approximated by

$$\widetilde{F}_e(\Omega) \approx \frac{i\sqrt{\Gamma_e}g_{om}}{\omega_0(i\Delta-\Gamma_t/2)}\left[a_0^*\delta\widetilde{A}(\Omega) + a_0\delta\widetilde{A}^*(-\Omega)\right]. \tag{S29}$$

In the case that the mechanical excitation is dominated by the external optical modulation, the thermal excitation from the reservoir is negligible and Eqs. (S12) and (S13) become

$$\frac{d^2x_b}{dt^2} + \Gamma_{mb}\frac{dx_b}{dt} + \Omega_{mb}^2 x_b + \frac{\kappa}{m_b}x_d = \frac{F_o}{m_b}, \tag{S30}$$

$$\frac{d^2x_d}{dt^2} + \Gamma_{md}\frac{dx_d}{dt} + \Omega_{md}^2 x_d + \frac{\kappa}{m_d}x_b = 0. \tag{S31}$$

Using Eqs. (S26) and (S27), following a similar procedure as the previous section, we find that the mechanical displacement for the optically bright mode is now given by

$$\widetilde{x}_b(\Omega) = \frac{\widetilde{F}_e(\Omega)}{m_b}\frac{L_d(\Omega)}{L_b(\Omega)L_d(\Omega)-\eta^4}, \tag{S32}$$

where $L_b(\Omega)$ and $L_d(\Omega)$ are given in the previous section. Clearly, the mechanical response given in Eq. (S32) is directly analogous to the atomic response in EIT systems [1].

V. CORRESPONDENCE OF CAVITY OPTOMECHANICS TO COHERENT STOKES AND ANTI-STOKES RAMAN SCATTERING

In this section, we show a direct correspondence between cavity optomechanics and coherent Stokes and anti-Stokes Raman scattering. The system Hamiltonian of an optomechanical cavity is given by the following general form:

$$\mathscr{H} = \hbar\omega_0 a^\dagger a + \hbar\Omega_m b^\dagger b + \hbar g_{om} x_b a^\dagger a, \tag{S33}$$

where a and b are the annihilation operators for photon and phonon, respectively, normalized such that $a^\dagger a$ and $b^\dagger b$ represent the operators for photon and phonon number. x_b is the mechanical displacement for the optically bright mode, related to b by

$$x_b = \sqrt{\frac{\hbar}{2m_b\Omega_{mb}}}\left(b+b^\dagger\right). \tag{S34}$$

 DOI: 10.1038/NPHOTON.2010.5

Therefore, the interaction Hamiltonian between the optical wave and the mechanical motion is given by

$$\mathscr{H}_i = \hbar g a^\dagger a \left(b + b^\dagger \right), \tag{S35}$$

where the factor $g \equiv \left(\frac{g_{om}^2 \hbar}{2 m_b \Omega_{mb}} \right)^{1/2}$.

The mechanical motion modulates the intracavity field to create two optical sidebands. As a result, the optical field can be written as

$$a = a_p + a_s e^{-i\Omega_{mb} t} + a_i e^{i\Omega_{mb} t}, \tag{S36}$$

where a_p is the field amplitude of the fundamental wave, and a_s and a_i are those of the generated Stokes and anti-Stokes wave, respectively. As the magnitudes of the Stokes and anti-Stokes sidebands are much smaller than the fundamental wave, when we substitute Eq. (S36) into Eq. (S35) and leave only the first-order terms of a_s and a_i, under the rotating-wave approximation, the interaction Hamiltonian becomes

$$\mathscr{H}_i = \hbar g \left(b + b^\dagger \right) a_p^\dagger a_p + \hbar g b^\dagger \left(a_s^\dagger a_p + a_p^\dagger a_i \right) + \hbar g b \left(a_p^\dagger a_s + a_i^\dagger a_p \right). \tag{S37}$$

In Eq. (S37), the first term describes the static mechanical actuation, which changes only the equilibrium position of mechanical motion and is neglected in the following analysis, as discussed previously. The second and third terms show clearly that the process corresponds directly to coherent Stokes and anti-Stokes Raman scattering as shown in Fig. 4d in the main text.

VI. THE MECHANICAL RESPONSE WITH THREE MODE COUPLING

The coherent mixing of mechanical excitation is universal to gradient-force-based NOMS with a giant optical spring effect. Similar phenomena to that presented for double-disks were also observed in the zipper cavity. However, due to its device geometry, the coupled nanobeam has more complex mechanical mode families in which all the even-order mechanical modes are optically dark, because they exhibit a mechanical node at the beam center where the optical mode is located. As the same-order common and differential motions of the two beams have similar mechanical frequencies, they can simultaneously couple to an optically bright mode, leading to multiple excitation interferences on the mechanical response.

In the case when the optically bright mode is coupled to two optically dark modes, the Hamiltonian for the mechanical system is given by the following general form:

$$\mathscr{H}_m = \sum_{i=b,1,2}\left(\frac{p_i^2}{2m_i}+\frac{1}{2}k_i x_i^2\right)+\kappa_1 x_b x_1+\kappa_2 x_b x_2, \tag{S38}$$

where $i = b, 1, 2$ corresponds to the optically bright mode and optically dark modes 1 and 2, respectively. With this Hamiltonian, counting in both the optical gradient force and the Langevin forces from the thermal reservoir, we obtain the equations of motions for the three modes:

$$\frac{d^2x_b}{dt^2}+\Gamma_{mb}\frac{dx_b}{dt}+\Omega_{mb}^2 x_b+\frac{\kappa_1}{m_b}x_1+\frac{\kappa_2}{m_b}x_2 = \frac{F_b}{m_b}+\frac{F_o}{m_b}, \tag{S39}$$

$$\frac{d^2x_1}{dt^2}+\Gamma_{m1}\frac{dx_1}{dt}+\Omega_{m1}^2 x_2+\frac{\kappa_1}{m_1}x_b = \frac{F_1}{m_1}, \tag{S40}$$

$$\frac{d^2x_2}{dt^2}+\Gamma_{m2}\frac{dx_2}{dt}+\Omega_{m2}^2 x_3+\frac{\kappa_2}{m_2}x_b = \frac{F_2}{m_2}, \tag{S41}$$

where the gradient force F_o is given by Eq. (S15), and the statistical properties of the Langevin forces are given by Eq. (S14). Following the same analysis as the previous section, we can obtain the spectral intensity for the mechanical displacement of the optically bright mode as

$$S_{x_b}(\Omega) = \frac{2k_BT}{m_b}\frac{\eta_1^4\Gamma_{m1}\left|L_2(\Omega)\right|^2+\eta_2^4\Gamma_{m2}\left|L_1(\Omega)\right|^2+\Gamma_{mb}\left|L_1(\Omega)L_2(\Omega)\right|^2}{\left|L_b(\Omega)L_1(\Omega)L_2(\Omega)-\eta_1^4L_2(\Omega)-\eta_2^4L_1(\Omega)\right|^2}, \tag{S42}$$

where $\eta_j^4 \equiv \frac{\kappa_j^2}{m_b m_j}$ $(j = 1, 2)$ represents the mechanical coupling coefficient. $L_j(\Omega) = \Omega_{mj}^2 - \Omega^2 - i\Gamma_{mj}\Omega$ $(j = 1, 2)$ and $L_b(\Omega)$ is given by Eq. (S20) with Ω'_{mb} and Γ'_{mb} given in Eqs. (S21) and (S22), respectively. As the optical wave is coupled to the optically bright mode only, the power spectral density of the cavity transmission is still given by Eq. (S9), with the mechanical response S_{x_b} given in Eq. (S42).

Figure S1 shows the PSD of the cavity transmission by launching a continuous wave into a resonance of the coupled nanobeams with an intrinsic and loaded Q factor of 3.0×10^4 and 2.8×10^4, respectively. Three mechanical modes are clearly visible, where mode I is the fundamental differential mode [Fig. S1(h)I], and mode II and III correspond to the second-order common and differential modes [Fig. S1(h)II and III], respectively. Similar to the double-disk NOMS, the gigantic optical spring effect shifts the frequency of the optically bright mode I from its intrinsic value of 8.06 MHz to 19 MHz, crossing over both optically dark modes II and III closely located at 16.54 and 17.04 MHz and resulting in complex interferences on the power spectra [Fig. S1(a)]. Equation (S42) provides an accurate description of the observed phenomena, as shown clearly

 DOI: 10.1038/NPHOTON.2010.5

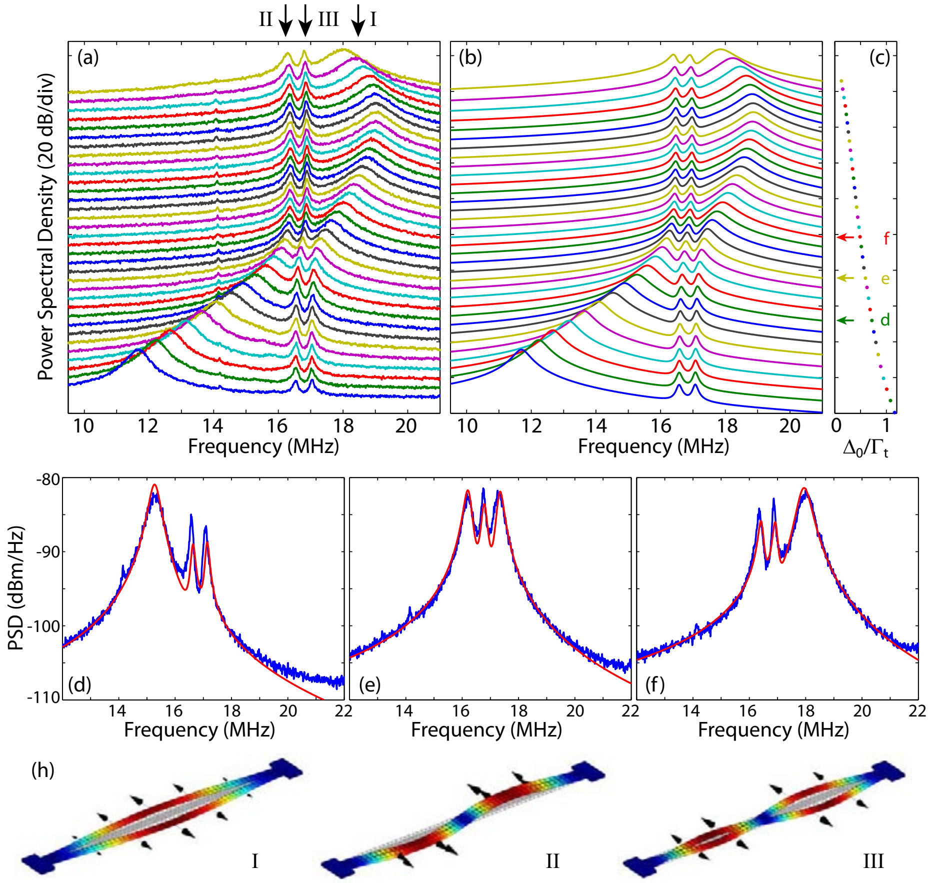

FIG. S1: (a) Experimentally recorded power spectral densities of the cavity transmission for the zipper cavity of Fig. 1d-f of the main text, with an input power of 5.1 mW. Each curve corresponds to a laser frequency detuning indicated in (c). Each curve is relatively shifted by 5 dB in the vertical axis for a better vision of the mechanical frequency tuning and the induced mechanical interference. The optically dark mode II and III have a full-width at half maximum (FWHM) of 0.16 and 0.15 MHz, respectively. The optically bright mode I has an intrinsic FWHM of 0.30 MHz. (b) The corresponding theoretical spectra of the power spectral density. (d)-(f) The detailed spectra of the power spectral density at three frequency detunings indicated by the three arrows in (c). The blue and red curves show the experimental and theoretical spectra, respectively. (h) FEM simulated mechanical motions for the fundamental *differential* mode (I), the second-order *common* (II) and *differential* (III) modes, whose frequencies are indicated by the arrows in (a). The color map indicates the relative magnitude (exaggerated) of the mechanical displacement.

in Fig. S1(b), (d)-(f). Fitting of the PSD results in mechanical coupling coefficients of $\eta_1/2\pi =$ 3.45 MHz and $\eta_2/2\pi = 3.48$ MHz, implying that the two optically dark modes couple to the fundamental optically bright mode with a similar magnitude.

VII. RENORMALIZATION OF MECHANICAL MODES BY THE GRADIENT FORCE

In addition to the gradient force and Langevin force, mechanical motion of the two microdisks (or two nanobeams) changes the gap between them and thus introduces a pressure differential between the gap and the outer region [2], which functions as a viscous force to damp the mechanical motion. In the previous sections, we have incorporated this squeeze-film effect in the mechanical damping rate. However, as the generated pressure differential is sensitive to the gap variations, we can expect that the squeeze-film effect behaves quite differently when the two disks/beams vibrate independently or cooperatively. Therefore, we describe it explicitly here for the analysis of mechanical mode renormalization.

In general, the motion of individual disks or nanobeams satisfies the following equations:

$$\frac{d^2x_1}{dt^2}+\Gamma_{m1}\frac{dx_1}{dt}+\Omega_{m1}^2x_1 = \frac{F_1}{m_1}+\frac{F_o}{m_1}+\frac{F_q}{m_1}, \tag{S43}$$

$$\frac{d^2x_2}{dt^2}+\Gamma_{m2}\frac{dx_2}{dt}+\Omega_{m2}^2x_2 = \frac{F_2}{m_2}-\frac{F_o}{m_2}-\frac{F_q}{m_2}, \tag{S44}$$

where F_q is the viscous force from the squeeze film damping, and m_j, x_j, Ω_{mj}, Γ_{mj}, F_j $(j=1,2)$ are the effective mass, the mechanical displacement, resonance frequency, damping rate, and the Langevin force for individual disks (or beams), respectively.

The optically bright mechanical mode corresponds to the differential motion of the two disks/beams, with a mechanical displacement given by $x_b \equiv x_1 - x_2$. By transferring Eqs. (S43) and (S44) into the frequency domain, it is easy to find that the mechanical displacement of the optically bright mode is given by

$$\widetilde{x}_b(\Omega) = \frac{\widetilde{F}_1(\Omega)}{m_1L_1(\Omega)} - \frac{\widetilde{F}_2(\Omega)}{m_2L_2(\Omega)} + \left[\frac{1}{m_1L_1(\Omega)}+\frac{1}{m_2L_2(\Omega)}\right]\left[\widetilde{F}_q(\Omega)+\widetilde{F}_o(\Omega)\right], \tag{S45}$$

where $L_j(\Omega) = \Omega_{mj}^2 - \Omega^2 - i\Gamma_{mj}\Omega$ $(j=1,2)$. The squeeze-film effect is produced by the pressure differential between the gap and the outer region introduced by the differential mechanical motion, and thus has a magnitude linearly proportional to the differential displacement. In general, it can be described by $\widetilde{F}_q(\Omega) = f_q(\Omega)\widetilde{x}_b(\Omega)$, where $f_q(\Omega)$ represents the spectral response of the squeeze gas film [2]. Using this form together with Eq. (S16) in Eq. (S45), we obtain the spectral intensity of the optically bright mode displacement,

$$S_{x_b}(\Omega) = \frac{2k_BT\left[\frac{\Gamma_{m1}}{m_1}|L_2(\Omega)|^2+\frac{\Gamma_{m2}}{m_2}|L_1(\Omega)|^2\right]}{\left|L_1(\Omega)L_2(\Omega)-\left[f_o(\Omega)+f_q(\Omega)\right]\left[\frac{L_1(\Omega)}{m_2}+\frac{L_2(\Omega)}{m_1}\right]\right|^2}. \tag{S46}$$

As the squeeze-film effect primarily damps the differential motion, its spectral response can be approximated as $f_q(\Omega) \approx i\alpha_q\Omega$. Moreover, since the two disks or nanobeams generally have only slight asymmetry due to fabrication imperfections, they generally have quite close effective masses and energy damping rates: $m_1 \approx m_2 = 2m_b$ and $\Gamma_{m1} \approx \Gamma_{m2} \equiv \Gamma_m$, where we have used the

 DOI: 10.1038/NPHOTON.2010.5

fact that the effective motional mass of the differential motion is given by $m_b = m_1 m_2/(m_1+m_2)$. As a result, Eq. (S46) can be well approximated by

$$S_{x_b}(\Omega) \approx \frac{k_B T \Gamma_m}{m_b} \frac{|L_1(\Omega)|^2 + |L_2(\Omega)|^2}{\left|L_1(\Omega)L_2(\Omega) - \frac{1}{2}\left[f_o(\Omega)/m_b + i\Gamma_q\Omega\right]\left[L_1(\Omega)+L_2(\Omega)\right]\right|^2}, \tag{S47}$$

where $\Gamma_q \equiv \alpha_q/m_b$ represents the damping rate introduced by the squeeze gas film, and the spectral response of the gradient force $f_o(\Omega)$ is given by Eq. (S16).

The intrinsic mechanical frequencies of 7.790 and 7.995 MHz for the two individual nanobeams are measured from the experimental recorded PSD with a large laser-cavity detuning. The optomechanical coupling coefficient is 68 GHz/nm and the effective mass is 10.75 pg for the fundamental differential mode, both obtained from FEM simulations (note that these values are different than those quoted in Ref. [3] due to the different definition of mode amplitude for x_b). The intrinsic and loaded optical Q factors are 3.0×10^4 and 2.8×10^4, respectively, obtained from optical characterization of the cavity resonance. By using these values in Eqs. (S47) and (S16), we can easily find the mechanical frequencies and linewidths for the two renormalized modes, where we treat the intrinsic mechanical damping rate Γ_m and the squeeze-film-induced damping rate Γ_q as fitting parameters. As shown in Fig. 2 of the main text, this theoretical model provides an accurate description of the mechanical mode renormalization, with a fitted intrinsic mechanical and squeeze-film damping rate of 0.03 and 0.2 MHz, respectively.

Similarly, we can obtain the spectral intensity of $x_d \equiv x_1 + x_2$ for the optically-dark mechanical mode, which is given by the following form:

$$S_{x_d}(\Omega) = 2k_B T \frac{\frac{\Gamma_{m2}}{m_2}\left|L_1(\Omega) - \frac{2}{m_1}\left[f_o(\Omega)+f_q(\Omega)\right]\right|^2 + \frac{\Gamma_{m1}}{m_1}\left|L_2(\Omega) - \frac{2}{m_2}\left[f_o(\Omega)+f_q(\Omega)\right]\right|^2}{\left|L_1(\Omega)L_2(\Omega) - \left[f_o(\Omega)+f_q(\Omega)\right]\left[\frac{L_1(\Omega)}{m_2}+\frac{L_2(\Omega)}{m_1}\right]\right|^2}. \tag{S48}$$

Similar to the optically-bright mode, with $m_1 \approx m_2 = 2m_b$ and $\Gamma_{m1} \approx \Gamma_{m2} \equiv \Gamma_m$, Eq. (S48) can be well approximated by

$$S_{x_d}(\Omega) \approx \frac{k_B T \Gamma_m}{m_d} \frac{|L_1(\Omega) - h(\Omega)|^2 + |L_2(\Omega) - h(\Omega)|^2}{\left|L_1(\Omega)L_2(\Omega) - \frac{1}{2}h(\Omega)\left[L_1(\Omega)+L_2(\Omega)\right]\right|^2}, \tag{S49}$$

where $m_d = m/2$ is the effective mass of the common mode and $h(\Omega) \equiv \left[f_o(\Omega)/m_b + i\Gamma_q\Omega\right]$ represents the total spectral response of the optical gradient force and squeeze film damping. In particular, when the optical-spring-induced frequency shift is much larger than the intrinsic mechanical frequency splitting, the spectral intensities of these two modes reduce to

$$S_{x_b}(\Omega) \approx \frac{2k_B T\Gamma_m/m_b}{|L_o(\Omega) - h(\Omega)|^2}, \qquad S_{x_d}(\Omega) \approx \frac{2k_B T\Gamma_m/m_d}{|L_o(\Omega)|^2} \tag{S50}$$

where $L_0(\Omega) = (\Omega_{m1} + \Omega_{m2})^2/4 - \Omega^2 - i\Gamma_m\Omega$. Equation (S50) indicates that the optically bright and dark modes reduce to a pure differential and common modes, respectively.

[1] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633–673 (2005).

[2] M. Bao and H. Yang, Sens. Actuators A **136**, 3–27 (2007).

[3] M. Eichenfield, R. Camacho, J. Chan, K. J. Vahala, and O. Painter, Nature **459**, 550–555 (2009).